Effects of ion temperature on electrostatic solitary structures in nonthermal plasmas

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Effects of ion temperature on compressive and rarefactive ion-acoustic solitary waves, which have been found to coexist in non-thermal plasmas, are investigated by the pseudopotential approach, which is valid for arbitrary amplitude solitary waves. It is shown that the effects of ion temperature change the minimum value of α (the parameter that determines the number of nonthermal electrons present in the plasma under consideration) as well as M (the Mach number) for which these solitary waves coexist and also change the width and amplitude of these solitary waves. It is also shown that for cold ions, the present results completely agree with the existing published results [Cairns *et al.*, Geophys. Res. Lett. **22**, 2709 (1995); J. Phys. (France) IV **5**, C6-43 (1995)]. [S1063-651X(97)05001-0]

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I. INTRODUCTION

The study of electrostatic solitary waves in plasmas has received considerable attention because of its vital role in understanding the nonlinear features of localized electrostatic disturbances in laboratory plasmas [1–3] as well as in space plasmas [4–6] and has been extensively studied in the past few years [7–11]. It is found theoretically and confirmed experimentally that if the ions are assumed to respond as a fluid to perturbations in the potential, with no significant trapping in a potential well, a thermal plasma supports only compressive solitary waves (solitary structures with density compression), but not rarefactive ones (solitary structures with density depletion). Recently, motivated by the observations of solitary structures with density depletions made by the Freja and Viking satellites [12,13], Cairns *et al.* [14,15] have considered a plasma consisting of nonthermally distributed electrons and cold ions and shown that it is possible to obtain both positive (compressive) and negative (rarefactive) solitary waves. As an extension and natural development of these investigations [14,15], the present work has considered the same plasma system, where ions are no longer cold, and studied the effects of ion temperature on the compressive and rarefactive solitary waves that have been found to coexist in this nonthermal plasma model. This paper is organized as follows. The basic equations are given in Sec. II. The ion temperature effects on one-dimensional solitary structures have been studied by the pseudopotential approach in Sec. III. This study has then been extended to three-dimensional structures in Sec. IV. Finally, a brief discussion is given in Sec. V.

II. GOVERNING EQUATIONS

We consider a plasma system consisting of warm adiabatic ions and nonthermally distributed electrons. The basic system of equations governing the ion dynamics in this plasma system is given by [16–18]

$$\frac{\partial n}{\partial t}+\nabla\cdot(n\mathbf{u})=0, \tag{1}$$

$$\frac{\partial u}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}=-\nabla\varphi-\frac{\sigma}{n}\nabla P, \tag{2}$$

$$\frac{\partial P}{\partial t}+\mathbf{u}\cdot\nabla P+\gamma P\nabla\cdot\mathbf{u}=0, \tag{3}$$

$$\nabla^2\varphi=n_e-n, \tag{4}$$

where n (n_e) is the ion (electron) density normalized to the unperturbed ion density n_0; $\sigma=T_i/T_e$, with T_i (T_e) being the ion (electron) temperature; u is the ion fluid velocity normalized to the ion-acoustic speed $C_s=(k_BT_e/m)^{1/2}$, with k_B and m being the Boltzmann constant and ion mass, respectively; P is the ion pressure normalized to $(n_0k_BT_i)$; $\gamma=(2+N)/N$, with N being the number of degrees of freedom (which has value 1 for the one-dimensional case and 3 for the three-dimensional case); φ is the electrostatic potential normalized to k_BT_e/e, with e being the electronic charge; the space variable is normalized to the Debye length $\lambda_D=(k_BT_e/4\pi n_0e^2)^{1/2}$ and the time variable is normalized to the ion plasma period $\omega_p^{-1}=(m/4\pi n_0e^2)^{1/2}$. As electrons are assumed to be nonthermally distributed, to model the electron distribution with a population of fast particles we can choose the distribution function as was chosen by Cairns *et al.* [14,15]. Therefore, without details, the electron density n_e in Eq. (4) is directly given by [14,15]

$$n_e=(1-\beta\varphi+\beta\varphi^2)e^{\varphi}, \tag{5}$$

$$\beta=\frac{4\alpha}{1+3\alpha}, \tag{6}$$

where α is a parameter determining the fast particles present in our plasma model.

III. ONE-DIMENSIONAL SOLITARY STRUCTURES

We will confine ourselves, in this section, to a study of solitary waves in our nonthermal plasma model for one-dimensional structures. The basic equations, in the one-dimensional case where $\gamma=3$, can be expressed as

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x}(nu)=0, \tag{7}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}=-\frac{\partial \varphi}{\partial x}-\frac{\sigma}{n}\frac{\partial P}{\partial x}, \tag{8}$$

$$\frac{\partial P}{\partial t}+u\frac{\partial P}{\partial x}+3P\frac{\partial u}{\partial x}=0, \tag{9}$$

$$\frac{\partial^2 \varphi}{\partial x^2}=(1-\beta\varphi+\beta\varphi^2)e^{\varphi}-n. \tag{10}$$

To obtain a solitary wave solution, we make all the dependent variables depend on a single independent variable $\xi = x - Mt$, where M is the Mach number (the velocity of the solitary wave normalized to the ion-acoustic speed C_s). Considering the steady-state condition, i.e., $\partial/\partial t = 0$, we can write our basic set of equations as

$$M\frac{\partial n}{\partial \xi}-\frac{\partial}{\partial \xi}(nu)=0, \tag{11}$$

$$M\frac{\partial u}{\partial \xi}-u\frac{\partial u}{\partial \xi}-\frac{\sigma}{n}\frac{\partial P}{\partial \xi}=\frac{\partial \varphi}{\partial \xi}, \tag{12}$$

$$M\frac{\partial P}{\partial \xi}-u\frac{\partial P}{\partial \xi}-3P\frac{\partial u}{\partial \xi}=0, \tag{13}$$

$$\frac{\partial^2 \varphi}{\partial \xi^2}=(1-\beta\varphi+\beta\varphi^2)e^{\varphi}-n. \tag{14}$$

Now, under the appropriate boundary conditions, viz., $\varphi \to 0$, $u \to 0$, $P \to 1$, and $n \to 1$ at $\xi \to \pm\infty$, Eqs. (11) and (13) can be integrated to give

$$n=\frac{1}{1-u/M}, \tag{15}$$

$$P=n^3. \tag{16}$$

If we substitute Eq. (15) into Eq. (12) and then multiply this by 2, we obtain

$$2M\frac{\partial u}{\partial \xi}-2u\frac{\partial u}{\partial \xi}-2\sigma\frac{\partial P}{\partial \xi}+2\frac{\sigma}{M}u\frac{\partial P}{\partial \xi}=2\frac{\partial \varphi}{\partial \xi}. \tag{17}$$

Again, multiplying Eq. (13) by σ/M one can write

$$\sigma\frac{\partial P}{\partial \xi}-\frac{\sigma}{M}u\frac{\partial P}{\partial \xi}-3P\frac{\sigma}{M}\frac{\partial u}{\partial \xi}=0. \tag{18}$$

Now, subtracting Eq. (17) from Eq. (18), one can get a differential equation that has the form

$$3\sigma\frac{\partial P}{\partial \xi}-3\frac{\sigma}{M}\frac{\partial}{\partial \xi}(Pu)-2M\frac{\partial u}{\partial \xi}+2u\frac{\partial u}{\partial \xi}+2\frac{\partial \varphi}{\partial \xi}=0. \tag{19}$$

The integration of this equation yields

$$3\frac{\sigma}{M}Pu-3\sigma(P-1)+2Mu-u^2-2\varphi=0, \tag{20}$$

where we have used the same boundary conditions, viz., $\varphi \to 0$, $u \to 0$, $P \to 1$, and $n \to 1$ at $\xi \to \pm\infty$. Substituting u and P, obtained from Eqs. (15) and (16), respectively, into this equation, one can obtain a quadratic equation for n^2 as

$$(3\sigma)n^4-(3\sigma+M^2-2\varphi)n^2+M^2=0. \tag{21}$$

Therefore, the solution of this equation for n is given by

$$n=\frac{\sigma_1}{\sqrt{2}\sigma_0}\left[1-\frac{2\varphi}{M^2\sigma_1^2}-\sqrt{\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)^2-4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2}, \tag{22}$$

where

$$\sigma_0=\sqrt{3\sigma/M^2}, \quad \sigma_1=\sqrt{1+\sigma_0^2}. \tag{23}$$

The substitution of this expression for n into Eq. (14) gives

$$\frac{d^2\varphi}{d\xi^2}=(1-\beta\varphi+\beta\varphi^2)e^{\varphi}-\frac{\sigma_1}{\sqrt{2}\sigma_0}\left[1-\frac{2\varphi}{M^2\sigma_1^2}-\sqrt{\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)^2-4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2}. \tag{24}$$

The qualitative nature of the solutions of this equation is most easily seen by introducing the Sagdeev potential [19]. Therefore, Eq. (24) takes the form

$$\frac{d^2\varphi}{d\xi^2}=-\frac{dV(\varphi)}{d\varphi}, \tag{25}$$

where the Sagdeev potential $V(\varphi)$ is given by

$$V(\varphi)=-[1+3\beta(1-\varphi)+\beta\varphi^2]e^{\varphi}-M^2\sqrt{\sigma_0}(e^{\theta/2}+\tfrac{1}{3}e^{-3\theta/2})+C_1, \tag{26}$$

$$\theta=\cosh^{-1}\left[\frac{\sigma_1^2}{2\sigma_0}\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)\right] \tag{27}$$

and C_1 is the integration constant that we will choose in such a manner that $V(\varphi)=0$ at $\varphi=0$. It is important to note here that we cannot consider the limit $\sigma \to 0$ in the Sagdeev potential $V(\varphi)$ in its present form. To consider this limit $\sigma \to 0$, we express θ as

$$\theta=\ln\left[\frac{\sigma_1^2}{2\sigma_0}\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)+\sqrt{\frac{\sigma_1^4}{4\sigma_0^2}\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)^2-1}\right]. \tag{28}$$

It is also important to mention that in our study the condition for ion density to be real, $|1-2(\varphi/M^2\sigma_1^2)| \geqslant 2\sigma_0/\sigma_1^2$, must always be satisfied. Using Eq. (28), we can express the Sagdeev potential $V(\varphi)$ as

$$V(\varphi)=-[1+3\beta(1-\varphi)+\beta\varphi^2]e^{\varphi} - \frac{M^2\sigma_1}{\sqrt{2}}\left[1-\frac{2\varphi}{M^2\sigma_1^2}+\sqrt{\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)^2-4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2} - \frac{2\sqrt{2}\sigma}{\sigma_1^3}\left[1-\frac{2\varphi}{M^2\sigma_1^2}+\sqrt{\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)^2-4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2} + C_1. \quad (29)$$

Now, we are again returning to our general equation, Eq. (25), which can be regarded as an ''energy law'' of an oscillating particle of unit mass with velocity $d\varphi/d\xi$ and position φ in a potential $V(\varphi)$. The solitary wave solutions of Eq. (25) exist if (i) $(d^2V/d\varphi^2)_{\varphi=0}<0$, so that the fixed point at the origin is unstable, and (ii) $V(\varphi)<0$ when $0<\varphi<\varphi_{\max}$ for positive solitary waves and $0>\varphi>\varphi_{\min}$ for negative solitary waves, where $\varphi_{\max\,(\min)}$ is the maximum (minimum) value of φ for which $V(\varphi)=0$. The general results can be obtained as follows. The nature of these solitary waves, whose amplitude tends to zero as the Mach number M tends to its critical value, can be found by expanding the Sagdeev potential to third order in a Taylor series in φ. The critical Mach number is that which corresponds to the vanishing of the quadratic term. At the same time, if the cubic term is negative, there is a potential well on the negative side and if the cubic term is positive, there is a potential well on the positive side. Therefore, by expanding the Sagdeev potential $V(\varphi)$, given by Eq. (29), around the origin the critical Mach number, at which the second derivative changes sign, can be found as

$$M_c=\sqrt{\frac{1}{2(1-\beta)}\,[1+\sqrt{1+12\sigma(1-\beta)}]} \quad (30)$$

and at this critical value of M the third derivative is negative, i.e., rarefactive solitary waves exist, if

$$S_\alpha<0,$$

$$S_\alpha=\frac{1}{2S_0^2}\left(1+19\frac{\sigma}{S_0}\right)-\frac{1}{6}, \quad (31)$$

$$S_0=\frac{1}{2(1-\beta)}\,[1+\sqrt{1+12\sigma(1-\beta)}].$$

This gives us a very simple criterion for analyzing the range of different parameters, viz., α and σ, for which the compressive and rarefactive types of solitary waves exist. It is obvious that the consideration of cold ion limit ($\sigma=0$) corresponds to our earlier work [14,15], where we have shown that the minimum value of α for which compressive and rarefactive solitary waves co-exist is $\sim$0.155 and that for $\alpha=0.2$ the critical Mach number (the minimum value of the Mach number above which the compressive and rarefactive solitary waves coexist) is $\sqrt{2}$. It is clear from Eqs. (29)–(31) that the Sagdeev potential $V(\varphi)$, the critical Mach number M_c (which we can now define as that minimum value of M above which compressive as well as rarefactive solitary waves exist), and S_α (which determines the criterion for the coexistence of compressive and rarefactive solitary waves) nonlinearly depend on σ, i.e., on ion temperature. Figure 1 shows how the minimum value of α, for which compressive and rarefactive solitary waves coexist, changes with σ, i.e., with ion temperature. This shows that as the ion temperature increases, we need more nonthermal electrons in order for rarefactive solitary waves to exist. We have already showed that for $\alpha=0.2$ the critical Mach number is $\sqrt{2}$, but due to the effect of the ion temperature this value changes. The plot in Fig. 2 shows how the critical Mach number changes with α and σ. It is clear that as the ion temperature increases, the critical Mach number (M_c) increases. It is observed that, for cold ions, i.e., $\sigma=0$, and for $\alpha=0.2$ (a value that we shall continue to use in the rest of our numerical illustrations), the compressive and rarefactive solitary waves are found to coexist when the Mach number passes the value $\sqrt{2}\simeq1.414$, but for $\sigma=0.02$, the rarefactive solitary waves do not exist until the Mach number exceeds the value 1.435. Figure 3 shows the behavior of the Sagdeev potential $V(\varphi)$ when the Mach number passes from 1.43 to 1.45. This shows that when the Mach number exceeds the value 1.435, a potential well forms on the negative φ axis, resulting in the existence of rarefactive solitary waves. To find what happens on the positive side, we plot curves for the same set of parameters on a larger scale. This is shown in Fig. 4, where it is seen that the compressive solitary waves also exist.

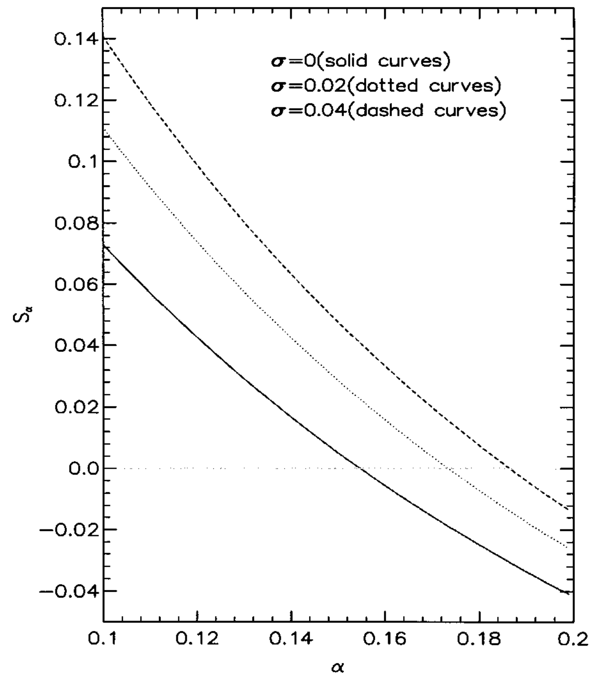

FIG. 1. Effect of ion temperature on the minimum value of α for which compressive and rarefactive solitary waves coexist.

Now, to see what happens when the ion temperature is further increased, we numerically study the behavior of the Sagdeev potential $V(\varphi)$ and find the parameters for which compressive and rarefactive solitary waves may coexist. These are displayed in Figs. 5 and 6. It is seen from Fig. 5 that when $\sigma=0.04$, rarefactive solitary waves no longer exist for $M=1.45$ (we have already found that, for values less than this, rarefactive solitary waves exist when $\sigma=0.02$), but when it exceeds this value, the rarefactive solitary waves start to exist. Figure 6, where the behavior of the Sagdeev

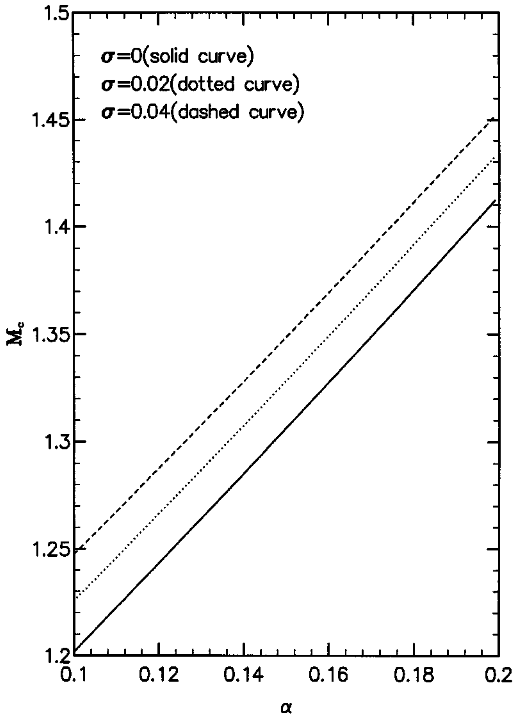

FIG. 2. Effect of ion temperature on the variation of the critical Mach number with α.

potential is shown for the same set of parameters on a larger scale, shows what happens on the positive φ axis. It is shown that as we increase the ion temperature, we need a higher Mach number in order to obtain the coexistence of compressive and rarefactive solitary waves. Figure 7 shows the effects of the ion temperature on potential profiles for two

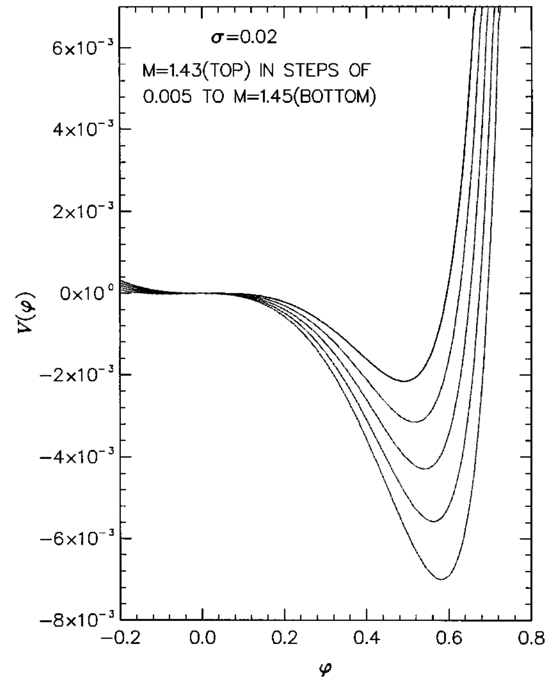

FIG. 4. Behavior of the Sagdeev potential $V(\varphi)$ on a larger scale for $\alpha=0.2$, $\sigma=0.02$, and a series of Mach numbers: 1.43 (top) in steps of 0.005 to 1.45 (bottom).

(compressive and rarefactive) solitary wave solutions found by solving Poisson's equation with exactly the same parameters, but different initial conditions. It is found that as the ion temperature increases, the amplitude of both the compressive and rarefactive solitary waves decreases, whereas their width increases.

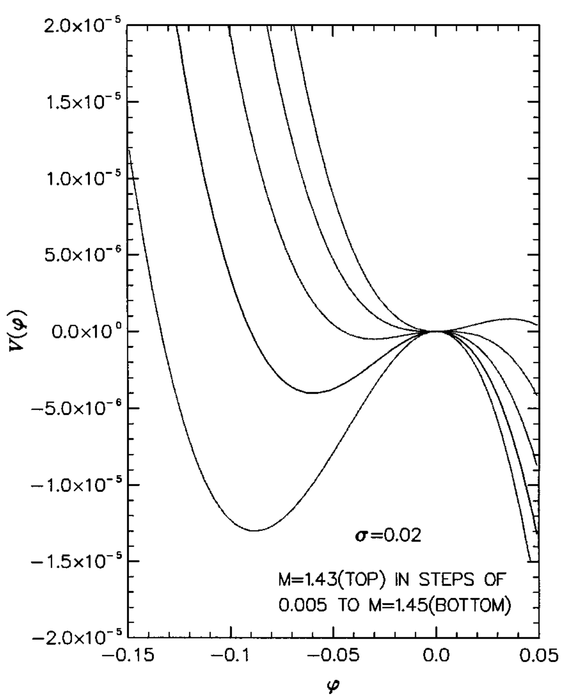

FIG. 3. Behavior of the Sagdeev potential $V(\varphi)$ for $\alpha=0.2$, $\sigma=0.02$, and a series of Mach numbers: 1.43 (top) in steps of 0.005 to 1.45 (bottom).

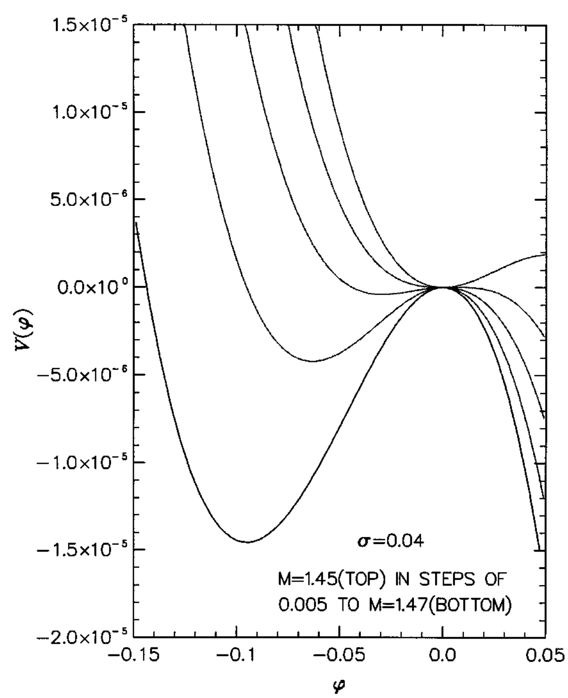

FIG. 5. Behavior of the Sagdeev potential $V(\varphi)$ for $\alpha=0.2$, $\sigma=0.04$, and a series of Mach numbers: 1.45 (top) in steps of 0.005 to 1.47 (bottom).

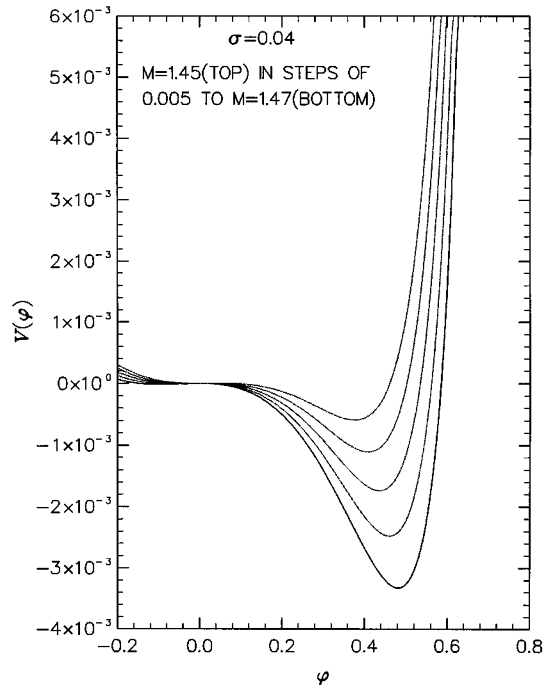

FIG. 6. Behavior of the Sagdeev potential $V(\varphi)$ on a larger scale for $\alpha=0.2$, $\sigma=0.04$, and a series of Mach numbers: 1.45 (top) of 0.005 to 1.47 (bottom).

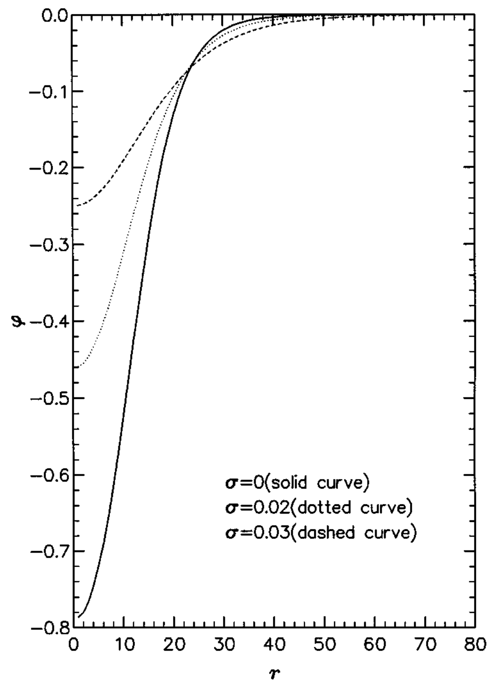

FIG. 8. Radial potential profiles for spherically symmetric solitary structures and for the same parameters used in Fig. 7, viz., $\alpha=0.2$, $M=1.455$, and $\sigma=0$ (solid curve), $\sigma=0.02$ (dotted curve), and $\sigma=0.03$ (dashed curve).

IV. THREE-DIMENSIONAL SOLITARY STRUCTURES

The solitary structures, discussed up to now, are one dimensional. In the present section, we will switch our attention to three-dimensional solitary structures, since the structures observed in space are certainly not infinite in two directions. A very simple three-dimensional analog of the structures, discussed in the preceding section, can be constructed by assuming that they are moving parallel to a strong magnetic field. If the ion Larmor radius is small compared to this size of the structure, we can just consider the ions to be a beam flowing along the field lines (in the rest frame of the structure). The ion density n then just depends on the potential, as before, and is given by Eq. (22). We also assume that the electrons have the same kind of adiabatic response and that the one-dimensional distribution, considered up to now, is obtained by integrating over the parallel degrees of freedom. Thus the electron density n_e is also the same and is directly given by Eq. (5). Therefore, under these assumptions, Poisson's equation, Eq. (4), can be expressed in the spherically symmetric case as

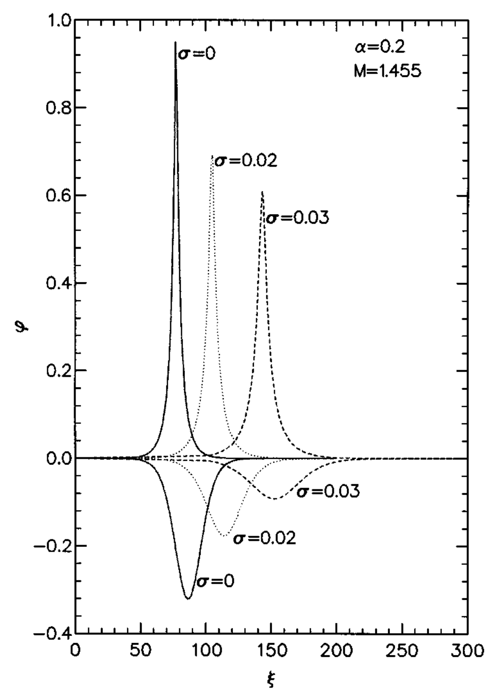

FIG. 7. Potential profiles for $\alpha=0.2$, $M=1.455$, and $\sigma=0$ (solid curve), $\sigma=0.02$ (dotted curve), and $\sigma=0.03$ (dashed curve).

$$\frac{d^2\varphi}{dr^2}+\frac{2}{r}\frac{d\varphi}{dr}=(1-\beta\varphi+\beta\varphi^2)e^{\varphi}-\frac{\sigma_1}{\sqrt{2}\sigma_0}\left[1-\frac{2\varphi}{M^2\sigma_1^2}-\sqrt{\left(1-\frac{2\varphi}{M^2\sigma_1^2}\right)^2-4\frac{\sigma_0^2}{\sigma_1^4}}\right]^{1/2}, \tag{32}$$

where the space variable r is normalized to the Debye length λ_D. We will now solve this equation numerically and find spherically symmetric structures that exist as solutions of this equation. It is important to note here that, in our numerical solutions of this equation, the condition for ion density to be real, $|1-2(\varphi/M^2\sigma_1^2)|\geqslant 2\sigma_0/\sigma_1^2$, must always be valid. The potential profiles in a spherically symmetric solution of this equation are illustrated in Fig. 8. These plots also show the effects of ion temperature on these radial profiles. The most obvious change, found by comparing these radial profiles with one-dimensional structures (discussed in Sec. III), is that there is a larger dip in the potential for the same parameters. It is also seen that as we increase the ion temperature, this dip in the potential decreases.

V. DISCUSSION

Motivated by the observations of solitary structures with density depletions made by the Freja and Viking satellites [12,13], Cairns *et al.* [14,15] have shown that the presence of nonthermal electrons changes the properties of the ion sound solitary waves and that for a suitable nonthermal electron distribution it is possible to obtain both positive (compressive) and negative (rarefactive) solitary waves. The present investigation is mainly concerned with ion temperature effects on these solitary structures. It is found that as we increase the ion temperature, we need more nonthermal electrons and higher Mach number in order for rarefactive solitary waves to exist. It has also been shown that as the ion temperature rises, the amplitude of both the compressive and rarefactive solitary waves decreases, whereas the width of these solitary waves increases. Three-dimensional structures have also been studied. The most obvious change is that there is a larger dip in the potential for the same parameters. It is also seen that as we increase the ion temperature, this dip in the potential decreases. It should be mentioned that for cold ions, the present analysis gives the same results as we have found in our earlier works [14,15].

This analysis may be of relevance to observations in the magnetosphere of density depressions [12,13]. A possible scenario is that lower hybrid turbulence produces, through modulational instability, cavities that collapse until the lower hybrid wave amplitude is sufficient to trap and accelerate a substantial number of electrons [10,11]. The damping of the turbulence could then leave a cavity and also create just the kind of energetic electron population necessary for it to live on as an ion-acoustic solitary structure no longer supported by the ponderomotive pressure of the high-frequency turbulence. However, the type of electron distribution we have looked at is common to many space and laboratory plasmas in which wave damping produces an electron tail, so the theory may be of more general interest.

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